The quadrature signal *s(t)* can be represented as:

$$s(t) = r(t) + jq(t)$$

where, r(t) is the real part of the signal *s(t):*

$$r(t) = A\cos(2\pi f_1 t + \phi) + n(t)$$

and *q(t)* is the quadrature part:

$$q(t) = A\sin(2\pi f_1 t + \phi) + \check{n}(t)$$

n(t) is the noise component and $\check{n}(t)$ is the Hilbert transform of *n(t)*.

With the Fourier analysis method the signal phase is measured using the following formula:

$$\phi_{es} = \tan^{-1}\left\{ \frac{\frac{1}{T}\int_0^T \left\{ -r(t)\sin(2\pi f_{es}t) + q(t)\cos(2\pi f_{es}t) \right\} dt}{\frac{1}{T}\int_0^T \left\{ r(t)\cos(2\pi f_{es}t) + q(t)\sin(2\pi f_{es}t) \right\} dt} \right\}$$

where T is the time over which the phase is measured. For simplicity, we will assume that the signal is clean (i.e, *n(t))* is zero) and $f_{es} = f_1$ (similar results will be obtained when the signal is noisy) and the measured phase is given by

$$\phi_{es} = \phi$$

This means that when complex signals are used and the signal is clean, the measured phase is exactly equal to the signal phase.

However, if the signal *s(t)* is represented by only the real part *r(t)*, then measured phase is given by,

$$\phi_{es} = \tan^{-1}\left\{ \frac{\frac{1}{T}\int_0^T \left\{ -r(t)\sin(2\pi f_{es}t) \right\} dt}{\frac{1}{T}\int_0^T \left\{ r(t)\cos(2\pi f_{es}t) \right\} dt} \right\}$$

Therefore, if only real signals are used, even if we assume that the signal is clean (i.e. *n(t)* = 0) and our estimate of the signal frequency is exact (i.e. $f_{es} = f_l$), the measured phase has two error factors and it is given by;

$$\phi_{es} = \tan^{-1}\left\{ \frac{\sin\phi + \frac{1}{T}\int_0^T (\sin(4\pi ft))\,dt}{\cos\phi + \frac{1}{T}\int_0^T \{\cos(4\pi ft)\}\,dt} \right\}$$

For the above equation, the two factors $\alpha = \frac{1}{T}\int_0^T \{\sin(4\pi ft)\}\,dt$ and $\beta = \frac{1}{T}\int_0^T \{\cos(4\pi ft)\}\,dt$ are the source of considerable error. For accurate phase measurement, both α and β should be zero. This condition can only be guaranteed when the integration time *T* is a multiple integer of 1/*f*. Figure (1) shows the error in the measured phase difference versus the integration time. It should be noted that these results also applied for the covariance processor.

For this plot, two signals are used. The phase of the first signal with respect to the reference signal is zero. Two cases are considered for the second signal. For these cases, the phases of the second signal with respect to the reference signal are 30^o and 60^o respectively.

From this plot, it is clear that to measure the phase accurately, the integration should be performed over an integer number of cycles. This condition can be guaranteed only when the signal is clean and the zero crossings of the Doppler signal are well defined. However, for noisy environments this task (i.e integration over an integer number of cycles) is difficult (if not impossible) to achieve. For these noisy environments, the only way to get reasonable phase measurement accuracy (say, error less than 2^o), is for the integration time to be greater than 10 cycles. Unfortunately, this limits the range of applications as the frequency dynamic range is affected. It should be pointed out that zero crossing based phase measurement methods suffer from this problem. With these methods, phase measurement is performed over an integer number of cycles. Accurate phase measurements cannot be attained at SNR less than 10 dB. Under these noisy conditions the location of the zero crossings is dominated by noise and not by the signal. Consequently, zero crossing methods fail to provide any meaningful results for noisy signals. These problems are not encountered when complex signals are used with the Fourier analysis method. Thus, for noisy environments, ***both the Fourier analysis method and complex signals must be used*** for optimum phase accuracy.

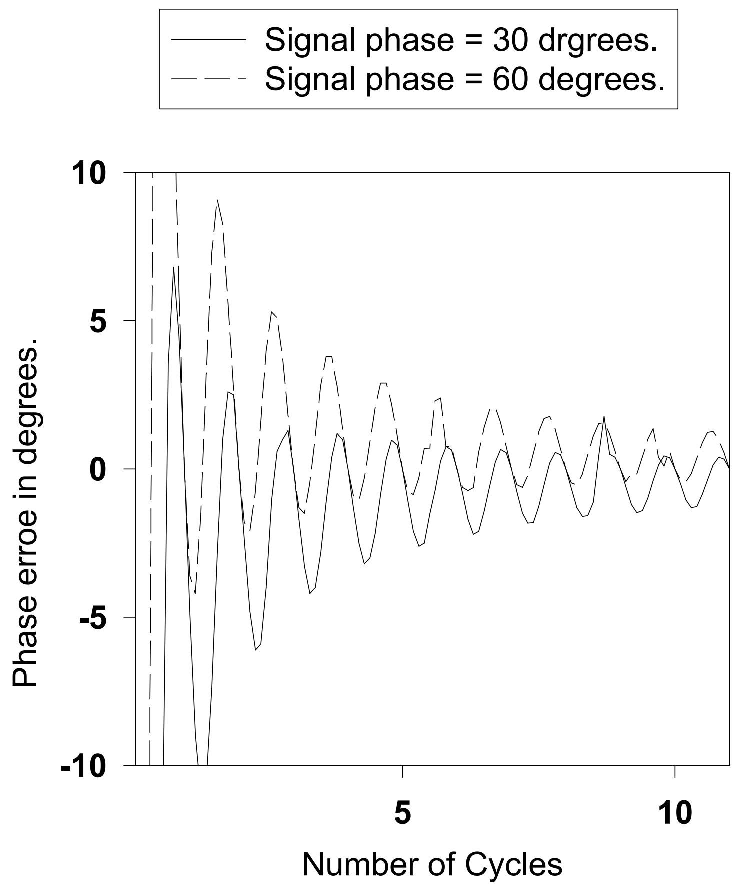

Fig.(1): signal phase error in degrees vs. measurement time in terms of numberof cycles.

Note: For complex signal, the phase error is zero everywhere except when the number of cycles=0 (at this point the phase is undefined).